coefficients of the planetary terms by about a fourth would also sensibly decrease the discordance between computation and observation.

(7) The consideration of the correction of Class III. (§ 3) is postponed for the present, pending further leisure. The value deduced for the "parallactic inequality" requires considerable correction, for the effect of systematic errors he has not taken into account.

(8) In conclusion I must congratulate Mr. Cowell on the degree of success which he has attained in dealing with the long series of Greenwich observations, and especially in showing so clearly that the theoretical values for the perturbations of the Moon by the Sun are in entire harmony with the results of observation. I can appreciate it all the more, as for years I had worked on the same subject; but my results, more detailed and elaborated than Mr. Cowell's, have for years been lying awaiting official publication, and will, I fear, have to wait for several years longer until the revenue of the colony is again normal; so Mr. Cowell's results take precedence, and mine must wait. I could have supplemented Mr. Cowell's data by furnishing the value derived from both observation and theory for most of the terms due to the planetary perturbations, but judged it preferable to keep entirely to the data furnished by Mr. Cowell's successful discussion of the observations.

Discussion of Greenwich Observations of the Sun, 1864–1900.

By P. H. Cowell, M.A.

An observation of a planet from the Earth being the exact counterpart of an observation of the Earth from the planet, it follows that the elements of the Earth's orbit can be obtained from observations of the planets. I hope to analyse the apparent errors of the Greenwich meridian observations of the planets for the years during which Le Verrier's tables were used in the *Nautical Almanac*, and I have already prepared such an analysis for the observations of *Venus*. The corrections to the elements of the Earth's orbit obtained directly from the Sun evidently form a useful check upon the results of the more complex discussion for a planet.

In this paper I only consider the right ascensions of the Sun, and I have only analysed for terms whose arguments are the longitude of the Earth and the difference of longitude of the Earth and *Venus*. In this way corrections to the solar eccentricity and perigee are obtained, and also a correction is obtained to the mass of *Venus*.

The figures to be analysed are given in Table I. The unit is $0^s.01$; the quantity tabulated is the mean error of right ascension for the month. Each month receives equal weight in the analysis, and the error for 1891 September has been supplied by interpolation.

TABLE I.

Mean Error of Sun's R.A. 1864–1900. *Unit* 0ˢ·01.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1864	− 2	− 4	− 4	+ 3	− 7	− 3	+ 4	+ 3	− 2	+ 3	+ 2	+ 7
1865	− 2	− 1	− 3	+ 3	+ 4	− 1	+ 2	+ 1	+ 10	+ 1	+ 4	+ 8
1866	− 4	0	− 4	− 1	− 7	− 1	0	+ 2	− 2	+ 5	+ 2	− 7
1867	− 8	−10	+ 1	− 5	− 3	− 7	+ 5	+ 2	0	0	0	+ 3
1868	− 6	− 5	− 6	− 3	− 3	+ 5	+ 2	+ 1	+ 10	+ 5	+ 9	− 3
1869	− 1	− 9	−13	0	+ 4	− 4	+ 6	+ 3	+ 6	+ 4	+ 3	+ 7
1870	− 3	− 5	− 3	− 2	− 1	− 1	+ 2	+ 1	+ 3	+ 6	+ 3	+ 11
1871	+ 2	0	− 3	− 3	+ 3	+ 3	+ 5	+ 1	+ 4	+ 1	+ 9	+ 3
1872	+ 2	− 3	− 9	− 5	− 4	0	+ 1	+ 1	+ 3	+ 4	+ 4	+ 2
1873	+ 2	+ 3	+ 3	− 3	+ 3	+ 5	− 1	+ 2	+ 7	+ 6	+ 6	+ 9
1874	+ 4	+ 4	+ 3	+ 1	+ 10	+ 6	+ 1	+ 6	+ 5	+ 8	+ 10	+ 9
1875	+ 8	+ 5	+ 3	− 3	+ 3	+ 10	+ 5	+ 2	+ 10	+ 6	+ 11	+ 4
1876	+ 2	+ 2	0	+ 2	+ 7	+ 7	+ 6	+ 7	+ 5	+ 3	+ 7	+ 4
1877	+ 5	0	− 2	− 6	+ 7	+ 5	+ 3	+ 4	− 1	+ 16	+ 8	+ 4
1878	+ 8	+ 2	+ 1	+ 1	− 2	+ 6	+ 9	+ 6	+ 5	+ 5	+ 12	+ 7
1879	+ 4	+ 3	+ 1	+ 4	− 1	+ 2	+ 6	+ 5	+ 3	+ 11	+ 10	+ 10
1880	+ 6	+ 4	− 5	0	+ 3	+ 11	+ 3	+ 6	+ 7	+ 5	+ 12	+ 19
1881	+ 6	+ 6	+ 3	0	+ 4	+ 3	+ 9	+ 5	+ 4	+ 7	+ 8	+ 4
1882	+ 7	+ 2	0	− 1	+ 1	+ 8	+ 2	+ 2	+ 3	+ 4	+ 8	+ 5
1883	+ 7	+ 4	+ 4	+ 2	+ 5	0	+ 6	+ 5	+ 4	+ 5	+ 6	+ 6
1884	+ 3	+ 1	+ 2	+ 2	+ 3	+ 12	+ 2	+ 8	+ 8	+ 6	+ 14	+ 12
1885	+ 9	− 4	+ 3	+ 5	+ 1	+ 7	+ 2	+ 1	+ 9	+ 8	+ 13	+ 7
1886	+ 10	+ 3	+ 2	+ 4	+ 4	+ 7	+ 8	+ 5	+ 9	+ 8	+ 12	+ 7
1887	+ 8	+ 5	+ 2	+ 4	+ 5	+ 4	+ 1	+ 9	+ 6	+ 5	+ 6	+ 8
1888	+ 5	+ 4	+ 6	0	+ 1	+ 3	+ 2	+ 2	+ 7	+ 4	+ 11	+ 7
1889	+ 7	+ 8	+ 5	+ 1	+ 3	+ 10	+ 4	+ 2	+ 9	+ 6	+ 13	+ 9
1890	+ 8	+ 5	+ 2	+ 4	− 3	0	+ 2	+ 8	+ 4	+ 4	+ 13	+ 11
1891	+ 7	+ 3	+ 6	+ 6	+ 5	+ 7	+ 8	+ 14	(+ 7)	+ 8	+ 14	+ 7
1892	+ 7	+ 3	0	+ 7	+ 6	+ 9	+ 10	+ 6	+ 7	+ 12	+ 16	+ 14
1893	+ 13	+ 8	+ 6	+10	+ 10	+ 7	+ 10	+ 9	+ 11	+ 7	+ 14	+ 5
1894	+ 12	+11	+ 8	+ 6	+ 4	+ 14	+ 13	+ 13	+ 17	+ 12	+ 16	+ 15
1895	+ 15	+11	+ 6	+ 7	+ 11	+ 10	+ 13	+ 10	+ 12	+ 12	+ 11	+ 12
1896	+ 10	+ 8	+ 6	+ 8	+ 8	+ 13	+ 9	+ 11	+ 10	+ 14	+ 16	+ 14
1897	+ 9	+ 8	+ 7	+ 4	+ 10	+ 10	+ 14	+ 14	+ 15	+ 17	+ 16	+ 10
1898	+ 12	+11	+ 5	+ 7	+ 11	+ 7	+ 8	+ 11	+ 16	+ 17	+ 14	+ 15
1899	+ 16	+ 5	+ 8	+ 8	+ 10	+ 9	+ 12	+ 13	+ 10	+ 17	+ 10	+ 17
1900	+ 11	+10	+ 9	+ 6	+ 6	+ 11	+ 8	+ 8	+ 11	+ 14	+ 16	+ 9
Sums	+199	+98	+50	+73	+121	+194	+202	+209	+252	+276	+359	+291

The sum of all the quantities in Table I. is +2324.

The tabular minus observed epoch is therefore $+23^{s}\cdot24 \div 444 = +0^{s}\cdot052 = +0''\cdot78$.

Analysing the sums for the different months and treating E, the Earth's longitude, as 115° for January, 145° for February, and so on, we get

$$\text{(tab.}-\text{obs.)} \; +0''\cdot78-0''\cdot47\sin(E-115^\circ)+0''\cdot00\cos(E-115^\circ) + (-0''\cdot21\sin+0''\cdot04\cos)2(E-115^\circ)$$

No appreciable part of the last term will correspond to possible errors of eccentricity or obliquity. The most probable explanation is instrumental error. The preceding term $-0''\cdot47 \sin(E-115^\circ)$ indicates that Le Verrier's eccentricity requires an increase. Here, again, instrumental error probably affects the result. We know, for instance, that there is a diurnal change in the level error whose amplitude is greater in summer than in winter. The effect of this could be roughly estimated, but it seems preferable to analyse the errors as they stand and merely point out that the results are liable to small systematic errors. The extent of the observations (thirty-seven years) is too short to obtain reliable corrections proportional to the time.

The corrections to Le Verrier's longitude in 1882 is therefore

$$-0''\cdot78+0''\cdot47\sin(E-115^\circ)$$
$$= -0''\cdot78+0''\cdot46\sin(E-100^\circ)-0''\cdot12\cos(E-100^\circ)$$

Element.	Correction required by Le Verrier in 1882.
Mean longitude	$-0''\cdot78$
Eccentricity	$+0\cdot23$
Coefficient of principal elliptic term in longitude	$+0\cdot46$
Mean anomaly	$-0\cdot12 \div 2e = -3''\cdot6$
Perihelion	$+2\cdot8$

It is interesting to compare these corrections with the differences between Le Verrier's tables and Newcomb's.

Counting T in centuries from 1900·0 we have the following differences, Newcomb–Le Verrier.

Element.	*Formula.* For Differences Newcomb–Le Verrier.	Value in 1882.
Mean longitude ...	$-0''\cdot71-0''\cdot82T-0''\cdot0183T$	$-0''\cdot56$
Perihelion	$+7\cdot8+17\cdot26T-0\cdot19T^2+0''\cdot012T^3$	$+4\cdot7$
Principal elliptic coefficient in longitude	$+0\cdot52+0\cdot27T+0\cdot004T^2$	$+0\cdot47$

There is most satisfactory agreement between the eccentricity found in this paper and that used by Newcomb. The mean longitude and perihelion cannot be compared unless allowance is made

(i) For any changes of long-period terms between Le Verrier's tables and Newcomb's.

(ii) For the difference $0''.8$ between Newcomb's equinox and that used in the Greenwich clock-star list up to and including 1902.

Coming now to the analysis with the heliocentric elongations of *Venus* as argument, an auxiliary angle with nineteen different values

$$180° + \frac{p \times 360°}{19}$$

was employed.

Approximately the value of p increases by unity in each month, that is to say, for each entry of Table I.; but in order to allow for the slight excess of the synodic period of *Venus* over nineteen Julian months, the value $p = 4$ is assigned to both 1868 January and February; $p = 14$ to both 1874 November and December; $p = 5$ to both 1885 September and October; $p = 15$ to 1894 July and August. We thus obtain

TABLE II.

V–E.	p.	Sum of Errors.	No.
198°.9	1	+106	23
217.9	2	+118	23
236.8	3	+104	23
255.8	4	+104	24
274.7	5	+138	24
293.7	6	+109	23
312.6	7	+121	23
331.6	8	+125	23
350.5	9	+130	23
9.5	10	+124	23
28.4	11	+143	23
47.4	12	+111	23
66.3	13	+127	24
85.3	14	+149	25
104.2	15	+128	25
123.2	16	+119	23
142.1	17	+ 97	23
161.1	18	+133	23
180.0	19	+138	23

Analysing in the usual way we have

$$(\text{tab.}-\text{obs.}) \quad +0''\cdot03 \sin (V-E)+0''\cdot00 \sin 2(V-E)$$

Newcomb, using a mass of *Venus* 1 : 408000, obtains terms in the Earth's longitude

$$-4''\cdot838 \sin (V-E)+5''\cdot526 \sin 2(V-E)$$

Le Verrier, using a mass of *Venus* 1 : 401847, in his theory obtains

$$-4''\cdot913 \sin (V-E)+5''\cdot613 \sin 2(V-E)$$

but in his tables he uses a mass (*Annales de l'Obs. de Paris*, vol. iv. p. 34) 1 : 400246 (p. 102) with inequalities

$$-4''\cdot93 \sin (V-E)+5''\cdot63 \sin 2(V-E)$$

To these terms the apparent corrections just obtained are

$$-0''\cdot03 \sin (V-E)+0''\cdot00 \sin 2(V-E)$$

For the mass of *Venus* we therefore have

from the term in	$V-E$	1·006	to 400246
„	„ $2(V-E)$	1·000	„ 400246
	Mean	1·003	to 400246.

Newcomb, in his provisional theory (*Astron. Constants*, pp. 7, 8), uses 1 : 401847, and on p. 22, last six lines, he obtains a correction +·007 from the Greenwich observations 1867–1892, in close accordance with the result of this paper. The present paper, however, supplements Newcomb's calculations by considering the two principal terms separately instead of all the terms together.

Professor Newcomb, however, swamps the result obtained from recent Greenwich observations by introducing additional material of decidedly inferior value, and on pp. 101–2 he reaches the result 1 —·0119 : 401847, and in his tables of the Sun he uses 1 : 408000, as stated above.

This latter value would imply that the two coefficients as obtained from the thirty-seven years 1864 to 1900 are respectively 0″·12 and 0″·10 in error. I do not believe that such errors are possible in the case of terms of short periods different from the periods of possible instrumental errors ; certainly in my analyses of the errors of the Moon I found no such discordances in cases where comparison could be made with well-determined theoretical values.

Moreover, in the older observations I did find large discordances in the case of the Moon, and I can easily believe that they may also exist in the case of the Sun. It must be remembered that the older observations of the Sun and Moon have not been

re-reduced with the same care that has lately been bestowed on several of the earlier star catalogues. I consider therefore that Professor Newcomb has given excessive weight to his other material, and that it would have been better to leave Le Verrier's mass of *Venus* unaltered.

Reciprocal of Mass of *Venus*.

Le Verrier	400×10^3
Newcomb	408×10^3
Present paper	399×10^3

Discussion of Greenwich Observations of Venus, 1869–1900.
By P. H. Cowell, M.A.

The observations under examination are divided into four periods of analysis of eight years each. Eight years is equivalent to thirteen revolutions of *Venus*, or five synodic periods. For each period of analysis corrections to the mean longitude, eccentricity, and perihelion of both the Earth and *Venus* are obtained. A correction of about 3″ to Le Verrier's centennial motion of *Venus* is distinctly indicated, but a period of thirty-two years is naturally too short to obtain reliable corrections proportional to the time. I have therefore merely taken means for the four periods of analysis, and thus obtained corrections applicable to Le Verrier's elements for 1885·0.

The peculiarity of my analysis is that it contains no simultaneous equations, a feature which I believe will be appreciated by readers of Le Verrier, vol. vi. pp. 58–70, or Newcomb, *Astronomical Constants*, p. 92. This has been achieved by forcing symmetry into the distribution of observations by giving enhanced weight, whenever necessary, to isolated observations. The process no doubt slightly diminishes the weight of the results as compared with an ideal method, but, as I shall now show, it involves a considerable increase of accuracy as compared with the more laborious methods just referred to. In Newcomb's *Astronomical Constants*, p. 57, we read: "In dealing with so extensive a system of unknown quantities it is impracticable to investigate the dependence of each upon all the others." Now it is just this dependence, which Newcomb found it impracticable to investigate, which is brought out in the present paper. And the investigation is by no means unnecessary; for I shall show that the corrections to the eccentricity and perihelion of the Earth are determined by a vector quantity whose components are $y+z$, $y'+z'$. A similar statement holds for *Venus* with components $y+2z$, $y'+2z'$. It also appears that y, y' are quantities that can only be determined with 10 per cent. of the weight